Let m and n be 4 and 2; or 27 and 1; or 9 and 3; etc.; then $2^3+1^2=3^2$; $3^3+13^2=14^2$; $3^3+3^2=6^2$; etc.

V. Solution by H. C. WILKES, Skull Run, West Virginia.

$x^3=(a+y)(a-y)$. Let $a+y=x^2$ and $a-y=x$, then $x^2+x=2a$, and $x=\frac{1}{2}\pm\sqrt{2a+\frac{1}{4}}$. Let a be any triangular number, and from the above formula, integral values for x, a, and y can be found.

VI. Solution by O. W. ANTHONY, M. Sc., Professor of Mathematics and Astronomy, New Windsor College, New Windsor, Maryland.

Let $x=ky$. Then $x^3+y^2=a^2$ becomes $y^2\{k^3y+1\}=a^2$. This will be a square if $y=k^3+2$. $\therefore$ $y=k^3+2$, and $x=k(k^3+2)$ will be a solution, where k is any integer. If $k=1$, $y=3$, $x=3$ and $x^3+y^2=36$. If $k=2$, $y=10$, $x=20$, and $x^3+y^2=8100$, etc., etc.

VII. Solution by J. H. DRUMMOND, LL. D., Portland, Maine.

(A). If the problem is to be taken literally, $y=\sqrt{a^2-x^3}$ in which x may any number whose third power $<$ than a^2. But this does not give exact results.

(B). If it means that $x^3+y^2=\square$, let $x=my$ and we have $m^3y+1=\square=$ (say) b^2 and $y=(b^2-1)/m^3$ and $x=(b^2-1)/m^2$; but then $a=b(b^2-1)/m^3$, in which m and b may be any numbers greater than unity, but the value of a depends on x and y.

(C). By transposing $x^3=a^2-y^2$; take $x=a-y$, then $x^2=a+y$, and $a^2-2ay+y^2=a+y$, and $y=(2a+1\pm\sqrt{8a+1})/2$. As y must be less than a to make x positive, the sign of the radical term must be negative. It is readily seen that $a=n(n+1)/2$ makes $8a+1$ a square, and by reducing we get $y=n(n-1)/2$ and $x=n$, in which n may be any number.

(D). If the question means to find exact values of x and y for *any* value of a, I cannot solve it.

46. Proposed by JOSIAH H. DRUMMOND, LL. D., Portland, Maine.

In $x^2+x\sqrt{xy}=a$......(1) and $y^2+y\sqrt{xy}=b$......(2) find such values of a find b as will make x and y integral; give a general solution.

I. Solution by the PROPOSER.

Take $y=m^2x$, and by combining the two equations and reducing we have, $\frac{b}{a}(m+1)=m^3(m+1)$ and consequently $m^3=\frac{b}{a}$.

From (1) we have $x=\pm\sqrt{\frac{a}{m+1}}$. Take $a=c^2$ and $m+1=d^2$ and substituting, we have $x=c/d$. To make this value integral, take $c=de$; then $x=e$, and $y=m^2x=e(d^2-1)^2$. But $a=c^2$, and $c=dx=de$. $\therefore$ $a=d^2e^2$; but $b=am^3=d^2e^2(d^2-1)^3$, in which a may be any whole number>1, and e any whole number.

II. **Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.**

In order that $\sqrt{xy}$ be integral and rational, we put $x=rm^2$ and $y=rn^2$, r, m, and n being any integers. Whence we readily find that when $a=r^2m^3(m+n)$ and $b=r^2n^3(m+n)$, x and y are integral.

Now put $r=1$, $m=3$, and $n=2$, and we obtain $x^2+x\sqrt{xy}=135$ and $y^2+y\sqrt{xy}=40$; whence $x=9$ and $y=4$.

Put $r=2$, $m=2$, and $n=1$; then $x^2+x\sqrt{xy}=96$ and $y^2+y\sqrt{xy}=12$; whence $x=8$, and $y=2$.

III. **Solution by A. H. BELL, Box 184, Hillsboro, Illinois.**

The only condition to fill is to make $xy=\square$. Take $x=4$, $y=1$, and $a=24$, $b=3$, etc., etc.

IV. **Solution by H. C. WILKES, Skull Run, West Virginia.**

Let $m^2=x$, $n^2=y$. Then $m^3(m+n)=a$; $n^3(m+n)=b$. $\therefore$ To make x and y integral, a and b must have a common factor $(m+n)$. The remaining factors will be m^3 and n^3. Let $a=448$, $b=189$; then $x=16$, $y=9$. $7(64)m=4$; $7(27)n=3$.

V. **Solution by G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas-Texas.**

Let $P=x^{\frac{1}{2}}$; $Q=y^{\frac{1}{2}}$. Then $P^4+P^3Q=a\ldots\ldots(1)$. $Q^4+Q^3P=b\ldots\ldots(2)$.

$$(1)\div(2),\ P=Q\sqrt[3]{a/b}.\quad \therefore P^2=x=\pm\frac{a^{\frac{2}{3}}}{\sqrt{a^{\frac{1}{3}}+b^{\frac{1}{3}}}},\ Q=y=\pm\frac{b^{\frac{2}{3}}}{\sqrt{a^{\frac{1}{3}}+b^{\frac{1}{3}}}}.$$

Let $a=\{\frac{1}{2}(m^2+n^2)\}^3$, $b=\{\frac{1}{2}(m^2-n^2)\}^3$.

$$\therefore x=\pm\frac{(m^2+n^2)^2}{4m},\quad y=\pm\frac{(m^2-n^2)^2}{4m}.$$

$$\text{Let } m=pn.\quad \therefore x=\pm\frac{n^3(p^2+1)^2}{4p},\quad y=\pm\frac{n^3(p^2-1)^2}{4p}.$$

Let $n=2p$. $\therefore x=\pm2p^2(p^2+1)^2$, $y=\pm2p^2(p^2-1)^2$.
$\therefore a=\{2p^2(p^2+1)\}^3$, $b=\{2p^2(p^2-1)\}^3$.

VI. **Solution by O. W. ANTHONY, M. Sc., Professor of Mathematics and Astronomy, New Windsor College, New Windsor, Maryland**

Let $y=m^2x$. Then $x^2(1+m)=a$, and $x^2m^3(1+m)=b$.

$$\therefore m=\frac{b^{\frac{1}{3}}}{a^{\frac{1}{3}}}.\quad x=\frac{a^{\frac{2}{3}}}{\sqrt{a^{\frac{1}{3}}+b^{\frac{1}{3}}}};\ y=\frac{b^{\frac{2}{3}}}{\sqrt{a^{\frac{1}{3}}+b^{\frac{1}{3}}}}.$$

$$\text{Let } a=p^3;\ b=q^3.\quad \text{Then } x=\frac{p^2}{\sqrt{p+q}};\ y=\frac{q^2}{\sqrt{p+q}}.$$

Let $p=2rs$; $q=r^2+s^2$. Then $x=\dfrac{4r^2s^2}{r+s}$; $y=\dfrac{(r^2+s^2)^2}{r+s}$.

Let $r=k+l$; $s=k-l$. Then $x=\dfrac{2(k^2-l^2)^2}{k}$; $y=\dfrac{2(k^2+l^2)^2}{k}$.

Let $l=\alpha k$. Then $x=2k^3(1-\alpha^2)^2$; $y=2k^3(1+\alpha^2)^2$.

Now $a=p^3=8r^3s^3=8(k^2-l^2)^3=8k^6(1-\alpha^2)^3$, and $b=q^3=(r^2+s^2)^3=8(k^2+l^2)^3=8k^6(1+\alpha^2)^3$, where α and k are integers.

PROBLEMS.

53. Proposed by A. H. BELL, Box 184, Hillsboro, Illinois.

Given $x^2-114\frac{1}{4}y^2=\mp3$ to find the least values of x and y in integers.

54. Proposed by JOSIAH H. DRUMMOND, LL. D., Portland, Maine.

In the expression $2x^2-2ax+b^2$, find two series of values for x in integral terms of a and b.

AVERAGE AND PROBABILITY.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

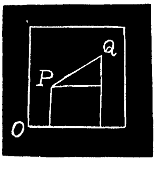
SOLUTIONS OF PROBLEMS.

35. Proposed by B. F. FINKEL, A. M., Professor of Mathematics and Physics in Drury College, Springfield, Missouri.

Find the chance that the distance of two points within a square shall not exceed a side of the square. [From *Byerly's Integral Calculus.*]

I. Solution by ALWYN C. SMITH, The University of Colorado, Boulder, Colorado.

a is one side of the square; P and Q the two points; (x, y) the point P with O for origin; and r and ϕ the polar coordinates of Q, with P as origin. Then the favorable cases are



$$4\int_0^{\frac{1}{2}\pi}\int_0^a\int_0^{a-r\sin\phi}\int_0^{a-r\cos\phi}dx\,dy\,r\,dr\,d\phi=a^4(\pi-\tfrac{13}{6}).$$

All the cases $=a^2.a^2=a^4$. Therefore, $p=\pi-\frac{13}{6}$.